Information-Theoretic 2-Round MPC without Round Collapsing: Adaptive Security, and More

Rachel Lin
University of Washington

Tianren Liu
University of Washington

Hoeteck Wee
NTT Research & ENS

TCC 2020

Information-Theoretic 2-Round MPC without Round Collapsing: Adaptive Security, and More

Simple 2-Round MPC

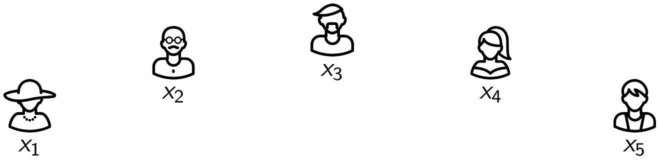
Rachel Lin
University of Washington

Tianren Liu
University of Washington

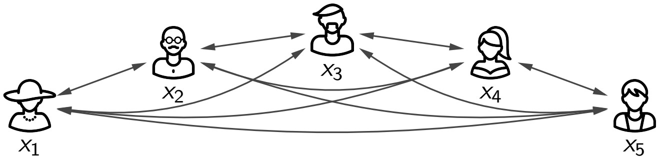
Hoeteck Wee
NTT Research & ENS

TCC 2020

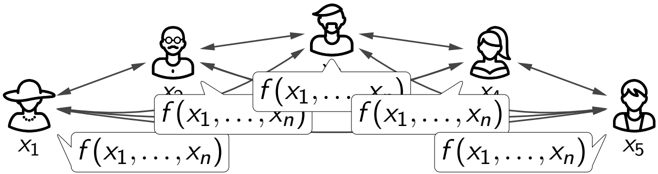
Multi-Party Computation



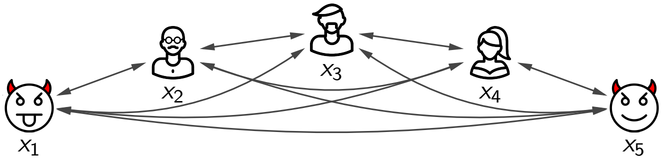
Multi-Party Computation



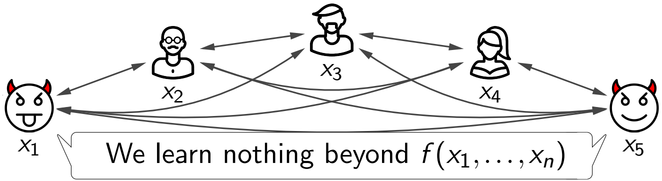
Multi-Party Computation



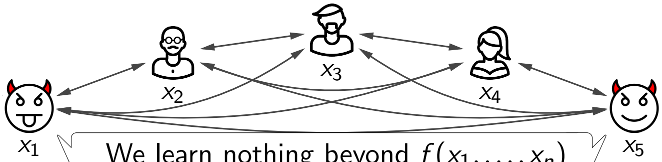
Multi-Party Computation



Multi-Party Computation



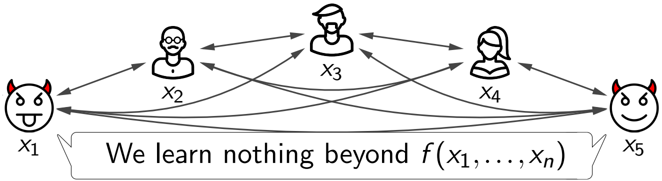
Multi-Party Computation



We learn nothing beyond $f(x_1, \ldots, x_n)$

*the corrupted parties' views can be simulated

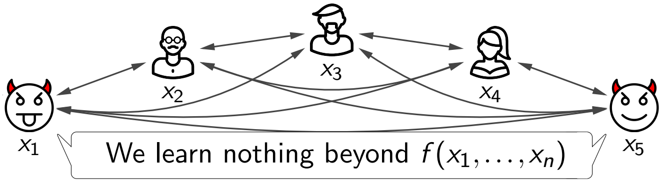
Multi-Party Computation



*the corrupted parties' views can be simulated

Adversary Semi-honest vs Malicious

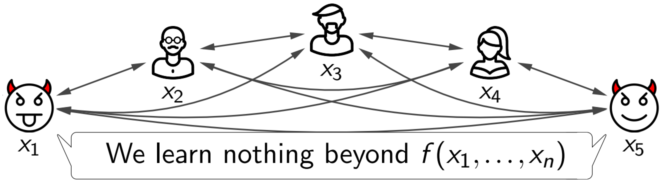
Multi-Party Computation



*the corrupted parties' views can be simulated

Adversary Semi-honest vs ~~Malicious~~

Multi-Party Computation

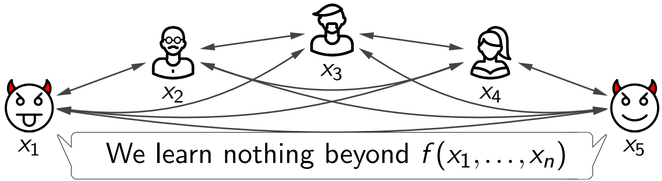


*the corrupted parties' views can be simulated

Adversary Semi-honest vs ~~Malicious~~

Corruption Static vs Adaptive

Multi-Party Computation

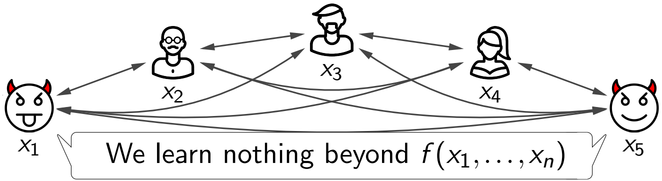


*the corrupted parties' views can be simulated

Adversary Semi-honest vs ~~Malicious~~

Corruption ~~Static~~ vs Adaptive

Multi-Party Computation



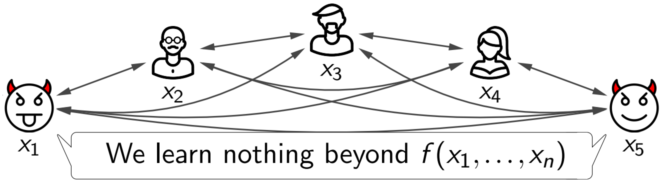
*the corrupted parties' views can be simulated

Adversary Semi-honest vs ~~Malicious~~

Corruption ~~Static~~ vs Adaptive

Security Computational vs Information-theoretic

Multi-Party Computation



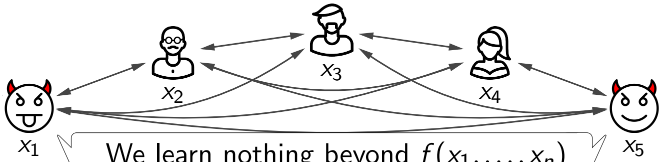
*the corrupted parties' views can be simulated

Adversary Semi-honest vs ~~Malicious~~

Corruption ~~Static~~ vs Adaptive

Security ~~Computational~~ vs Information-theoretic (NC1)

Multi-Party Computation



We learn nothing beyond $f(x_1, \ldots, x_n)$

*the corrupted parties' views can be simulated

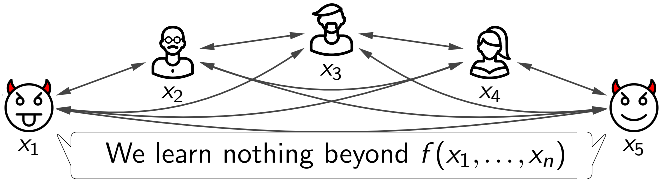
Adversary Semi-honest vs ~~Malicious~~

Corruption ~~Static~~ vs Adaptive

Security ~~Computational~~ vs Information-theoretic (NC1)

Computation Boolean vs Arithmetic

Multi-Party Computation



*the corrupted parties' views can be simulated

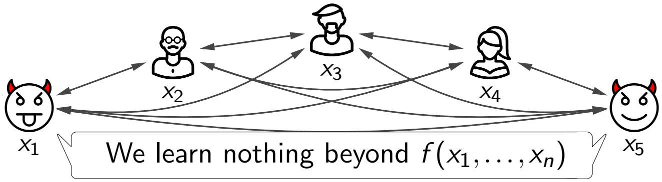
Adversary Semi-honest vs ~~Malicious~~

Corruption ~~Static~~ vs Adaptive

Security ~~Computational~~ vs Information-theoretic (NC1)

Computation ~~Boolean~~ vs Arithmetic (Black-box field)

Multi-Party Computation



*the corrupted parties' views can be simulated

Adversary Semi-honest vs ~~Malicious~~

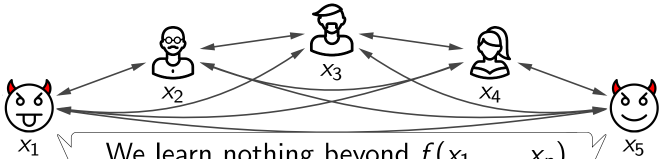
Corruption ~~Static~~ vs Adaptive

Security ~~Computational~~ vs Information-theoretic (NC1)

Computation ~~Boolean~~ vs Arithmetic (Black-box field)

Model

Multi-Party Computation



We learn nothing beyond $f(x_1, \ldots, x_n)$

*the corrupted parties' views can be simulated

Adversary Semi-honest vs ~~Malicious~~

Corruption ~~Static~~ vs Adaptive

Security ~~Computational~~ vs Information-theoretic (NC1)

Computation ~~Boolean~~ vs Arithmetic (Black-box field)

Model Plain model, Correlated randomness model

honest majority — honest minority

Optimal Round Complexity of Semi-Honest MPC

Honest majority	Honest minority

Optimal Round Complexity of Semi-Honest MPC

Honest majority	Honest minority
[Yao,BGW,DI,BMR] [IK00,IK02] 3-round & info-theoretic	

Optimal Round Complexity of Semi-Honest MPC

Honest majority

[Yao,BGW,DI,BMR]

[IK00,IK02]

3-round & info-theoretic

Honest minority

3 rounds

2 rounds

Optimal Round Complexity of Semi-Honest MPC

	Honest majority		Honest minority
3 rounds	[Yao,BGW,DI,BMR] [IK00,IK02] 3-round & info-theoretic		
2 rounds			[GGHR14] assume iO

Optimal Round Complexity of Semi-Honest MPC

	Honest majority		Honest minority
3 rounds	[Yao,BGW,DI,BMR] [IK00,IK02] 3-round & info-theoretic		
2 rounds			[GGHR14] assume iO [MW16] MKFHE+CRS

Optimal Round Complexity of Semi-Honest MPC

Honest majority

[Yao,BGW,DI,BMR]
[IK00,IK02]
3-round & info-theoretic

3 rounds

2 rounds

Honest minority

[GGHR14]
assume iO

[MW16]
MKFHE+CRS

[BL18,GS18]
2-round OT

Optimal Round Complexity of Semi-Honest MPC

Honest majority		Honest minority
[Yao,BGW,DI,BMR] [IK00,IK02] 3-round & info-theoretic	3 rounds	
	2 rounds	[GGHR14] assume iO [MW16] MKFHE+CRS [BL18,GS18] 2-round OT [BLPV18, GIS18,IMO18]

Optimal Round Complexity of Semi-Honest MPC

	Honest majority		Honest minority
3 rounds	[Yao,BGW,DI,BMR] [IK00,IK02] 3-round & info-theoretic		
2 rounds	[ACGJ18] assume OWF		[GGHR14] assume iO [MW16] MKFHE+CRS [BL18,GS18] 2-round OT [BLPV18, GIS18,IMO18]

Optimal Round Complexity of Semi-Honest MPC

	Honest majority		Honest minority
3 rounds	[Yao,BGW,DI,BMR] [IK00,IK02] 3-round & info-theoretic		
2 rounds	[ACGJ18] assume OWF		[GGHR14] assume iO
	[ABT18,GIS18] info-theoretic		[MW16] MKFHE+CRS
			[BL18,GS18] 2-round OT
			[BLPV18, GIS18,IMO18]

Optimal Round Complexity of Semi-Honest MPC

Honest majority		Honest minority
[Yao,BGW,DI,BMR] [IK00,IK02] 3-round & info-theoretic	3 rounds	
[ACGJ18] assume OWF [ABT18,GIS18] info-theoretic	2 rounds	[GGHR14] assume iO [MW16] MKFHE+CRS [BL18,GS18] 2-round OT [BLPV18, GIS18,IMO18]

Our Results

Honest majority	Honest minority

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1 tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions in plain model	

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

$r_A, s_A \in \mathbb{F}$ $r_B, s_B \in \mathbb{F}$

s.t. $r_A r_B = s_A + s_B$.

i.e. random r_A, r_B & additive share of $r_A r_B$

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

$r_A, s_A \in \mathbb{F}$ $\quad$ $r_B, s_B \in \mathbb{F}$

s.t. $r_A r_B = s_A + s_B$.

i.e. random r_A, r_B & additive share of $r_A r_B$

arithmetic analog of OT correlation

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1 tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions in plain model	info-theoretic 2-round MPC for NC1 tolerating $n-1$ corruptions in OLE correlated randomness model

Key Contribution: Simplicity

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

Key Contribution: Simplicity

- new adaptive security w/ explicit adaptive simulator*

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

Key Contribution: Simplicity

- new adaptive security w/ explicit adaptive simulator*
- new support arithmetic NC1 w/ black-box field access

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

Key Contribution: Simplicity

- $^{\text{new}}$adaptive security w/ explicit adaptive simulator*
- $^{\text{new}}$support arithmetic NC1 w/ black-box field access
- more efficient

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

Key Contribution: Simplicity

- new adaptive security w/ explicit adaptive simulator*
- new support arithmetic NC1 w/ black-box field access
- more efficient

extension to P/poly with black-box use of PRG

Our Results

Honest majority	Honest minority
info-theoretic 2-round MPC for NC1	info-theoretic 2-round MPC for NC1
tolerating $\lfloor \frac{n-1}{2} \rfloor$ corruptions	tolerating $n-1$ corruptions
in plain model	in OLE correlated randomness model

Key Contribution: Simplicity

- new adaptive security w/ explicit adaptive simulator*
- new support arithmetic NC1 w/ black-box field access
- more efficient

extension to P/poly with black-box use of PRG

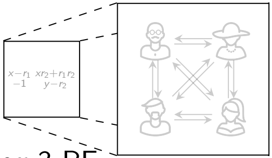
Key Technique: Direct Construction w/o Round Collapsing

What is Round Collapsing (here we illustrate the honest majority variant)

$$\begin{pmatrix} x - r_1 & x r_2 + r_1 r_2 \\ -1 & y - r_2 \end{pmatrix}$$

deg-3 RE
for NC1

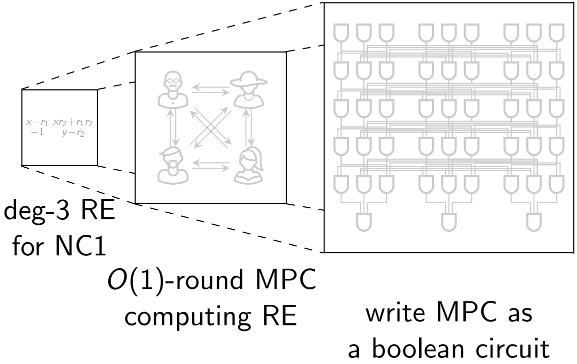
What is Round Collapsing (here we illustrate the honest majority variant)



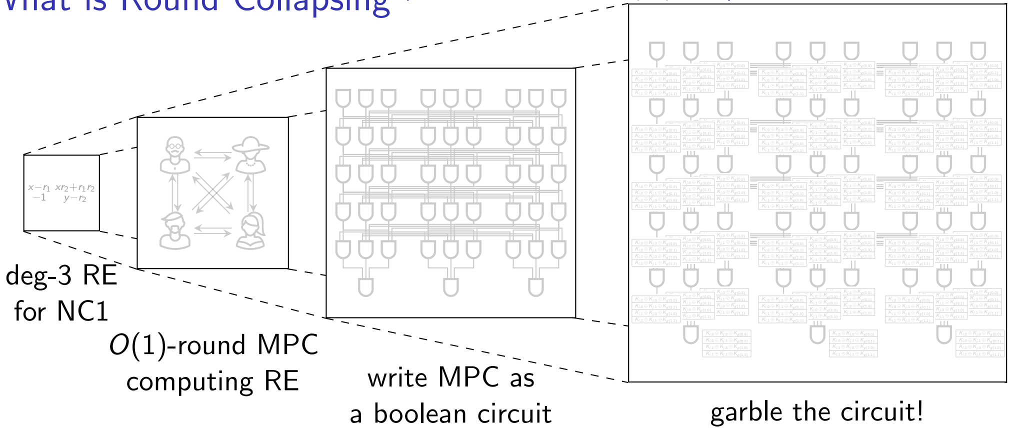
deg-3 RE
for NC1

$O(1)$-round MPC
computing RE

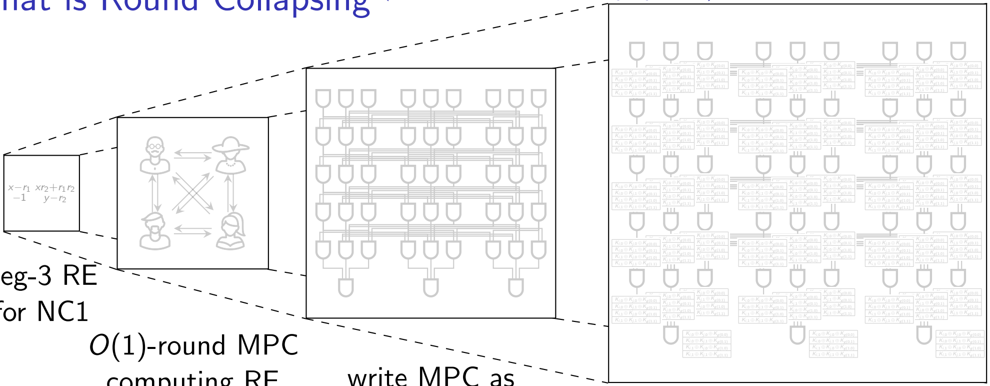
What is Round Collapsing (here we illustrate the honest majority variant)



What is Round Collapsing (here we illustrate the honest majority variant)



What is Round Collapsing (here we illustrate the honest majority variant)

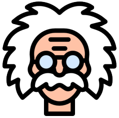


deg-3 RE
for NC1

$O(1)$-round MPC
computing RE

write MPC as
a boolean circuit

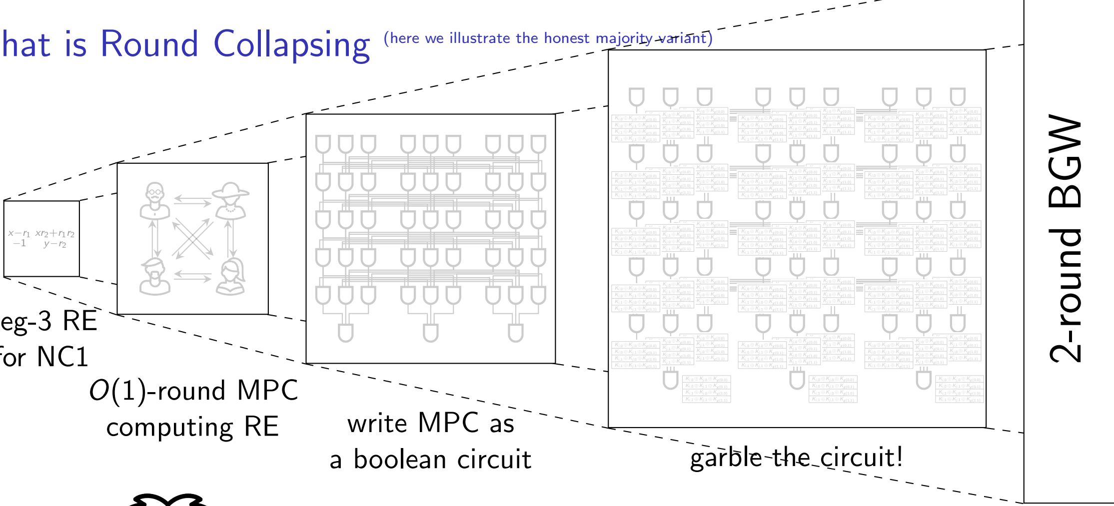
garble the circuit!

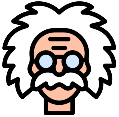


[ABT18]

The garbled circuit is
"effectively" degree-2
on *preprocessed*
local input and randomness

What is Round Collapsing (here we illustrate the honest majority variant)



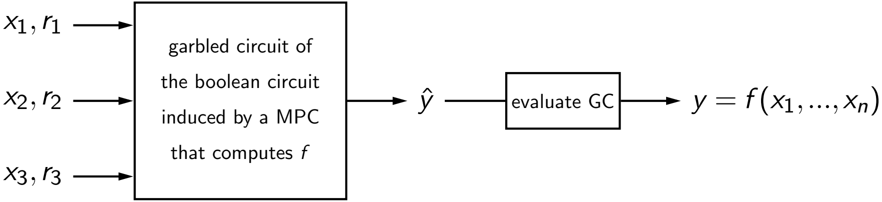


[ABT18]

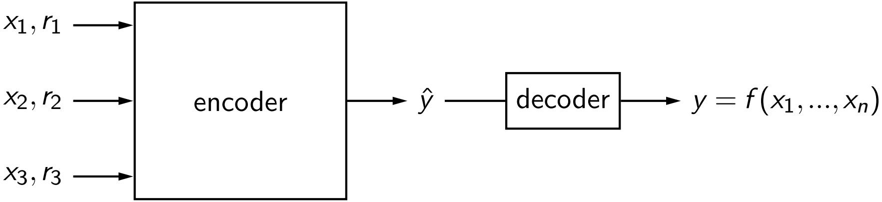
The garbled circuit is *"effectively"* degree-2 on *preprocessed* local input and randomness

The garbled circuit can be computed by 2-round BGW assume honest major

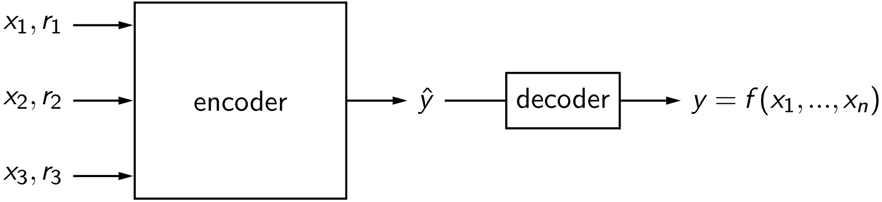
Multi-Party Randomized Encoding (MPRE) [ABT18]



Multi-Party Randomized Encoding (MPRE) [ABT18]

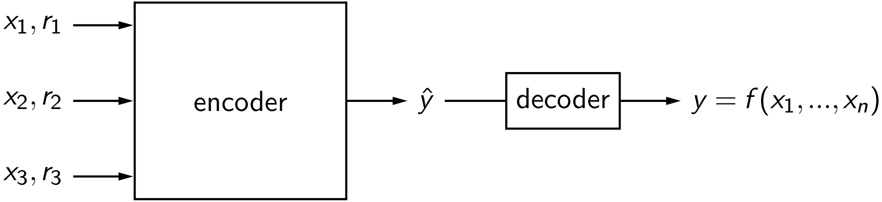


Multi-Party Randomized Encoding (MPRE) [ABT18]



Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

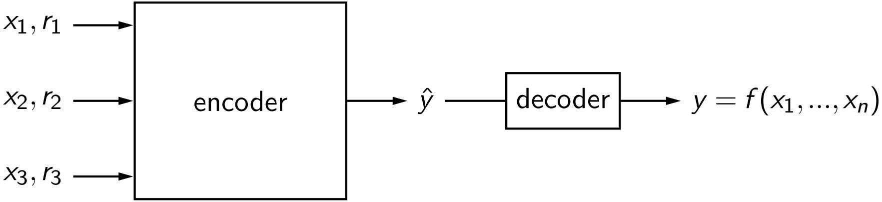
Multi-Party Randomized Encoding (MPRE) [ABT18]



Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

Privacy $\hat{y} \overset{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n))$

Multi-Party Randomized Encoding (MPRE) [ABT18]

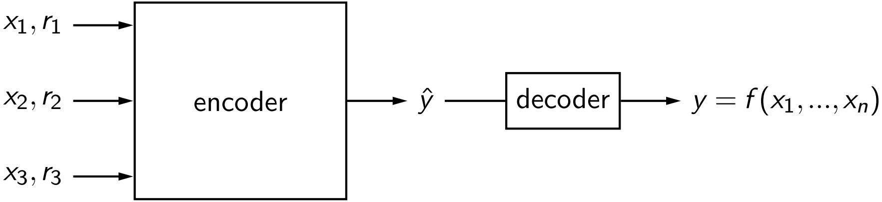


Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

Privacy $\hat{y} \stackrel{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \stackrel{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$

Multi-Party Randomized Encoding (MPRE) [ABT18]

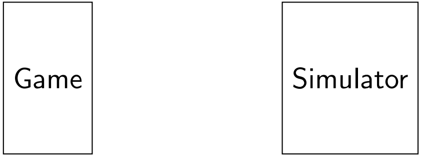


Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

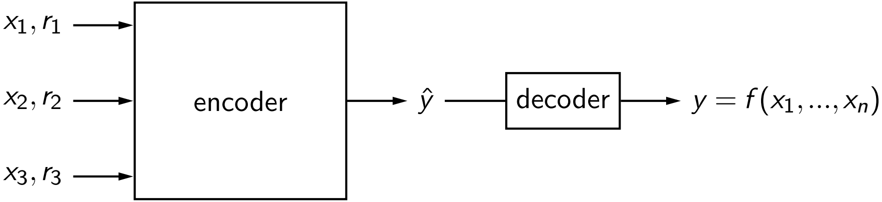
Privacy $\hat{y} \overset{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \overset{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$

Adaptive t-Privacy



Multi-Party Randomized Encoding (MPRE) [ABT18]

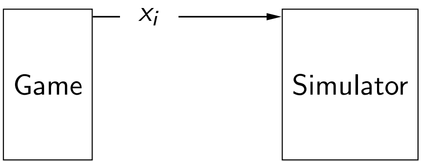


Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

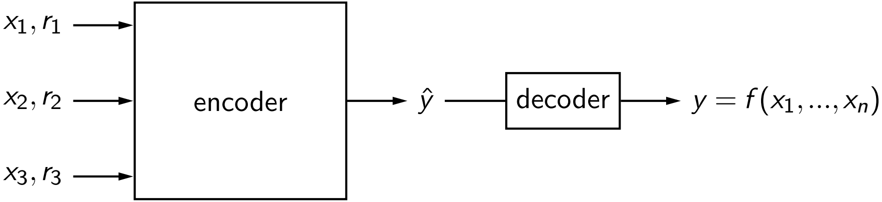
Privacy $\hat{y} \stackrel{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \stackrel{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$

Adaptive t-Privacy



Multi-Party Randomized Encoding (MPRE) [ABT18]

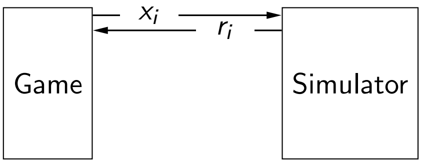


Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

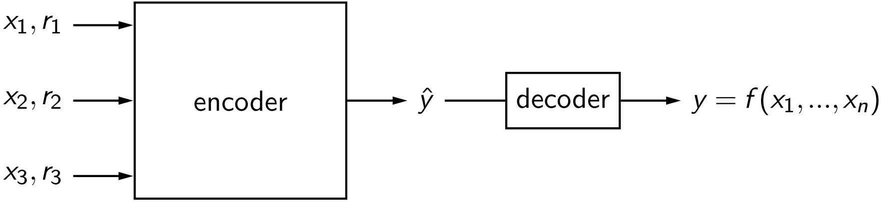
Privacy $\hat{y} \overset{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \overset{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$

Adaptive *t*-Privacy



Multi-Party Randomized Encoding (MPRE) [ABT18]

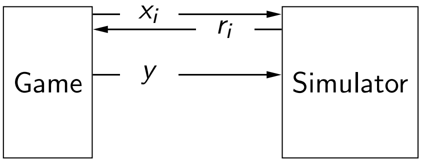


Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

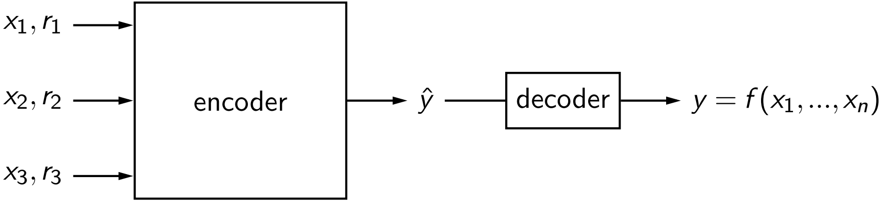
Privacy $\hat{y} \stackrel{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \stackrel{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$

Adaptive t-Privacy



Multi-Party Randomized Encoding (MPRE) [ABT18]

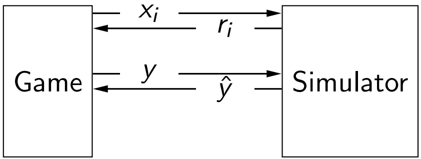


Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

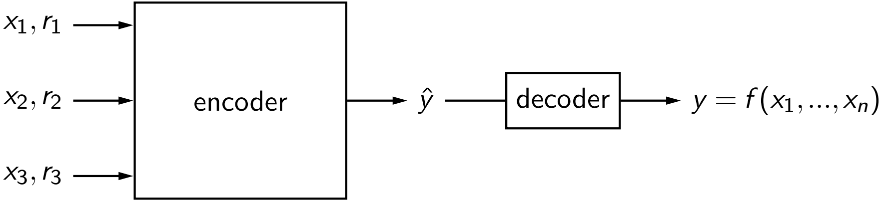
Privacy $\hat{y} \stackrel{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n))$

***t*-Privacy** $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \stackrel{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$

Adaptive *t*-Privacy



Multi-Party Randomized Encoding (MPRE) [ABT18]

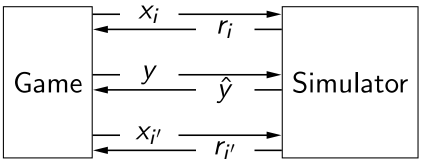


Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

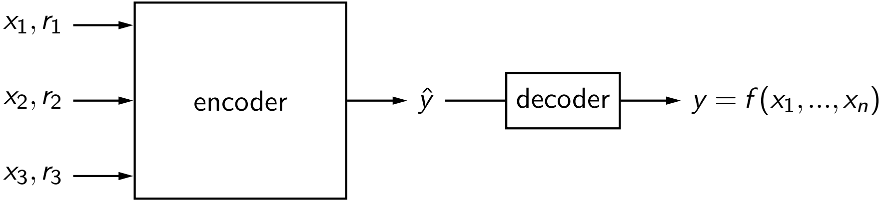
Privacy $\hat{y} \stackrel{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \stackrel{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$

Adaptive t-Privacy



Multi-Party Randomized Encoding (MPRE) [ABT18]

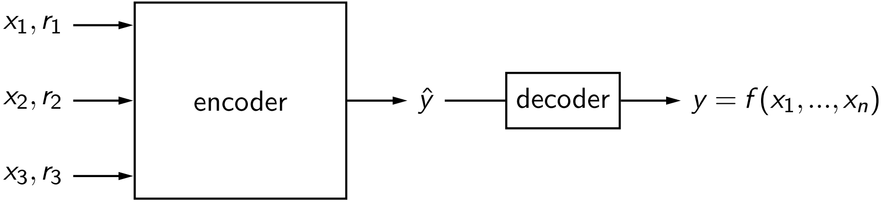


Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

Privacy $\hat{y} \stackrel{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \stackrel{d}{=} \mathrm{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$

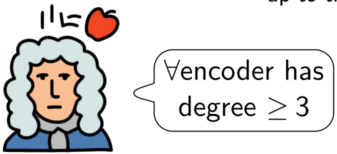
Multi-Party Randomized Encoding (MPRE) [ABT18]



Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

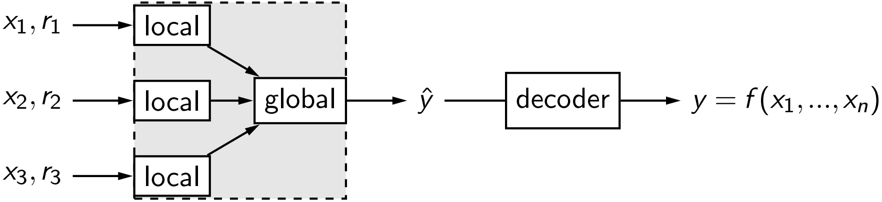
Privacy $\hat{y} \overset{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \overset{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$



[IK00]

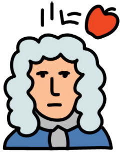
Multi-Party Randomized Encoding (MPRE) [ABT18]



Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

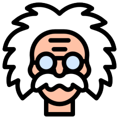
Privacy $\hat{y} \overset{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n))$

t-Privacy $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \overset{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$



$\forall$encoder has degree ≥ 3

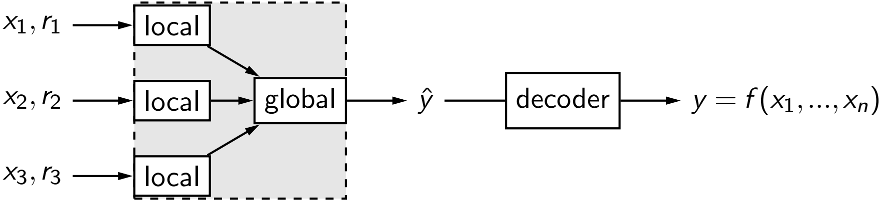
local pre-processing & global computation



[IK00]

[ABT18]

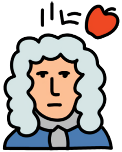
Multi-Party Randomized Encoding (MPRE) [ABT18]



Correctness $\hat{y}$ is decoded to $f(x_1, \ldots, x_n)$

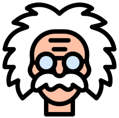
Privacy $\hat{y} \overset{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n))$

***t*-Privacy** $(\hat{y}, \underbrace{r_{i_1}, \ldots, r_{i_t}}_{\text{up to threshold } t}) \overset{d}{=} \mathsf{Sim}(f(x_1, \ldots, x_n), x_{i_1}, \ldots, x_{i_t})$



$\forall$encoder has degree ≥ 3

$\exists$encoder has degree 2 after pre-processing



[IK00]

[ABT18]

Multi-Party Randomized Encoding (MPRE) [ABT18]

Theorem [ABT18]: degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Multi-Party Randomized Encoding (MPRE) [ABT18]

Theorem [ABT18]

degree-2 MPRE for NC1 $+$ 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Proof.

x_1 x_2 x_3 x_4 x_5

Multi-Party Randomized Encoding (MPRE) [ABT18]

Theorem [ABT18]

degree-2 MPRE for NC1 $+$ 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

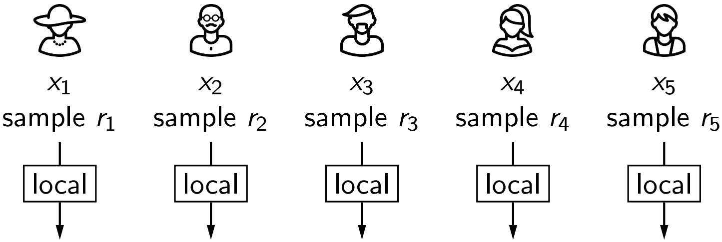
Proof.

x_1	x_2	x_3	x_4	x_5
sample r_1	sample r_2	sample r_3	sample r_4	sample r_5

Multi-Party Randomized Encoding (MPRE) [ABT18]

Theorem [ABT18]: degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

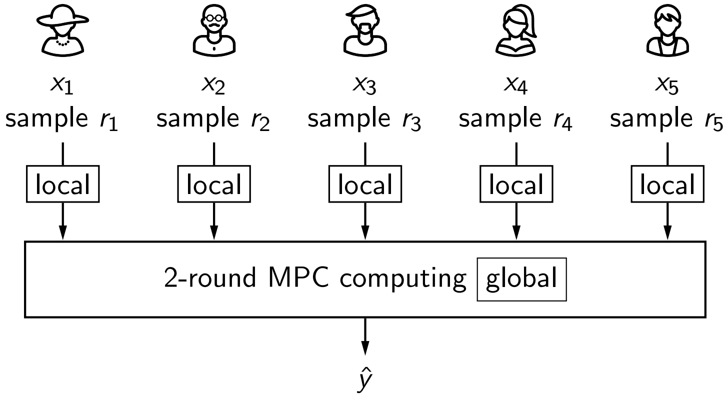
Proof.



Multi-Party Randomized Encoding (MPRE) [ABT18]

Theorem [ABT18]: degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

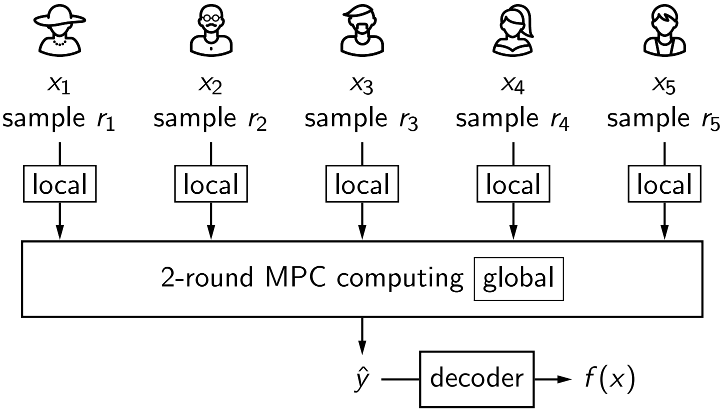
Proof.



Multi-Party Randomized Encoding (MPRE) [ABT18]

Theorem [ABT18]: degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Proof.



The Rest of the Talk...

Theorem [ABT18]

degree-2 MPRE for NC1 $+$ 2-round MPC for degree-2 poly $\implies$ 2-round MPC for NC1

The Rest of the Talk...

Theorem [ABT18]

degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Honest majority

The Rest of the Talk...

Theorem [ABT18]: degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Honest majority

$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1

The Rest of the Talk...

Theorem [ABT18]: degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Honest majority

$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1

$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model

The Rest of the Talk...

Theorem [ABT18]

degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Honest majority

$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1

$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model

BGW

The Rest of the Talk...

Theorem [ABT18]: degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Honest majority	Honest minority
$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	
$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model	

BGW

The Rest of the Talk...

Theorem [ABT18]

degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Honest majority	Honest minority
$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	$(n-1)$-private degree-2 MPRE for NC1 using OLE correlated randomness
$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model	

BGW

The Rest of the Talk...

Theorem [ABT18]	degree-2 MPRE for NC1	+	2-round MPC for degree-2 poly	$\Longrightarrow$	2-round MPC for NC1

Honest majority	Honest minority
$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	$(n-1)$-private degree-2 MPRE for NC1 using OLE correlated randomness
BGW	$(n-1)$-private 2-round MPC for degree-2 polynomial using OLE correlated randomness

The Rest of the Talk...

Theorem [ABT18] degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Honest majority	Honest minority
$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	$(n-1)$-private degree-2 MPRE for NC1 using OLE correlated randomness
BGW	≈arithmetic analog of passive-secure GMW

The Rest of the Talk...

Theorem [ABT18]: degree-2 MPRE for NC1 + 2-round MPC for degree-2 poly $\Longrightarrow$ 2-round MPC for NC1

Honest majority	Honest minority
$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	$(n-1)$-private degree-2 MPRE for NC1 using OLE correlated randomness
$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model	$(n-1)$-private 2-round MPC for degree-2 polynomial using OLE correlated randomness
BGW	$\approx$arithmetic analog of passive-secure GMW

The Rest of the Talk...

Theorem [ABT18] deg... ⟹ 2-round MPC for NC1

Breif review of
IK randomized encoding for NC1

Honest majority	Honest minority
$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	$(n-1)$-private degree-2 MPRE for NC1 using OLE correlated randomness
$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model	$(n-1)$-private 2-round MPC for degree-2 polynomial using OLE correlated randomness
BGW	≈arithmetic analog of passive-secure GMW

The Rest of the Talk...

Theorem [ABT18]

de

1

Breif review of
IK randomized encoding for NC1

$\Longrightarrow$

2-round MPC for NC1

Honest majority	Honest minority
$\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	$(n-1)$-private degree-2 MPRE for NC1 using OLE correlated randomness
$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model	$(n-1)$-private 2-round MPC for degree-2 polynomial using OLE correlated randomness

BGW

≈arithmetic analog of passive-secure GMW

The Rest of the Talk...

Theorem [ABT18] | deg... | ⟹ | 2-round MPC for NC1

1 Breif review of IK randomized encoding for NC1

Honest majority	Honest minority
2 $\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	$(n-1)$-private degree-2 MPRE for NC1 using OLE correlated randomness
$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model (BGW)	$(n-1)$-private 2-round MPC for degree-2 polynomial using OLE correlated randomness (≈arithmetic analog of passive-secure GMW)

The Rest of the Talk...

Theorem [ABT18] deg... ⟹ 2-round MPC for NC1

① Breif review of IK randomized encoding for NC1

Honest majority	Honest minority
② $\lfloor \frac{n-1}{2} \rfloor$-private degree-2 MPRE for NC1	③ $(n-1)$-private degree-2 MPRE for NC1 using OLE correlated randomness
$\lfloor \frac{n-1}{2} \rfloor$-private 2-round MPC for degree-2 polynomial in plain model (BGW)	$(n-1)$-private 2-round MPC for degree-2 polynomial using OLE correlated randomness (≈arithmetic analog of passive-secure GMW)

Briefly Recall IK Randomized Encoding

Briefly Recall IK Randomized Encoding

Any NC1 function f can be evaluated as a determinant

$$f(\vec{x}) = \det \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix}$$

linear in $\vec{x}$

Briefly Recall IK Randomized Encoding

Any NC1 function f can be evaluated as a determinant

$$f(\vec{x}) = \det \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix}$$

linear in $\vec{x}$

Example: $xyz + s = \det \begin{bmatrix} x & & s \\ -1 & y & \\ & -1 & z \end{bmatrix}$

Briefly Recall IK Randomized Encoding

Any NC1 function f can be evaluated as a determinant

$$f(\vec{x}) = \det \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix}$$

linear in $\vec{x}$

Briefly Recall IK Randomized Encoding

Any NC1 function f can be evaluated as a determinant

$$f(\vec{x}) = \det \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix}$$

linear in $\vec{x}$

randomized encoding

IK02

$$RE = \begin{bmatrix} 1 & s_2 & s_3 & s_4 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix} \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} \\ & 1 & r_{23} & r_{24} \\ & & 1 & r_{34} \\ & & & 1 \end{bmatrix}$$

randomness

linear in $\vec{x}$

randomness

Briefly Recall IK Randomized Encoding

Any NC1 function f can be evaluated as a determinant

$$f(\vec{x}) = \det \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix}$$

linear in $\vec{x}$

randomized encoding

IK02

$$RE = \begin{bmatrix} 1 & s_2 & s_3 & s_4 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix} \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} \\ & 1 & r_{23} & r_{24} \\ & & 1 & r_{34} \\ & & & 1 \end{bmatrix}$$

randomness

linear in $\vec{x}$

randomness

properties

1) $\det(RE) = f(\vec{x})$

Briefly Recall IK Randomized Encoding

Any NC1 function f can be evaluated as a determinant

$$f(\vec{x}) = \det \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix}$$

linear in $\vec{x}$

randomized encoding

IK02

$$\mathrm{RE} = \begin{bmatrix} 1 & s_2 & s_3 & s_4 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix} \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} \\ & 1 & r_{23} & r_{24} \\ & & 1 & r_{34} \\ & & & 1 \end{bmatrix}$$

randomness

linear in $\vec{x}$

randomness

properties

1) $\det(\mathrm{RE}) = f(\vec{x})$

2) $\mathrm{RE} \stackrel{d}{=} \mathrm{Sim}(f(\vec{x}))$

Briefly Recall IK Randomized Encoding

Any NC1 function f can be evaluated as a determinant

$$f(\vec{x}) = \det \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix}$$

linear in $\vec{x}$

properties

1) $\det(\mathrm{RE}) = f(\vec{x})$
2) $\mathrm{RE} \overset{d}{=} \mathrm{Sim}(f(\vec{x}))$
3) RE is arithmetic

randomized encoding

IK02

$$\mathrm{RE} = \begin{bmatrix} 1 & s_2 & s_3 & s_4 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix} \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} \\ & 1 & r_{23} & r_{24} \\ & & 1 & r_{34} \\ & & & 1 \end{bmatrix}$$

randomness

linear in $\vec{x}$

randomness

Briefly Recall IK Randomized Encoding

Any NC1 function f can be evaluated as a determinant

$$f(\vec{x}) = \det \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix}$$

linear in $\vec{x}$

randomized encoding

IK02

$$RE = \begin{bmatrix} 1 & s_2 & s_3 & s_4 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} L_{1,1}(\vec{x}) & L_{1,2}(\vec{x}) & L_{1,3}(\vec{x}) & L_{1,4}(\vec{x}) \\ -1 & L_{2,2}(\vec{x}) & L_{2,3}(\vec{x}) & L_{2,4}(\vec{x}) \\ & -1 & L_{3,3}(\vec{x}) & L_{3,4}(\vec{x}) \\ & & -1 & L_{4,4}(\vec{x}) \end{bmatrix} \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} \\ & 1 & r_{23} & r_{24} \\ & & 1 & r_{34} \\ & & & 1 \end{bmatrix}$$

randomness — linear in $\vec{x}$ — randomness

properties

1) $\det(RE) = f(\vec{x})$

2) $RE \stackrel{d}{=} \mathrm{Sim}(f(\vec{x}))$

3) RE is arithmetic

4) RE has degree 3

Direct Construction of degree-2 MPRE

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P y, s_2, Q z, s_3

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P $\quad$ y, s_2, Q $\quad$ z, s_3

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$
- PQ is a degree-$(n-1)$ poly,

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P y, s_2, Q z, s_3

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$
- PQ is a degree-$(n-1)$ poly,
  $xy = (PQ)(0) = \text{linear}((PQ)(1), \ldots, (PQ)(n))$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P $\quad$ y, s_2, Q $\quad$ z, s_3

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$
- PQ is a degree-$(n-1)$ poly,
  $xy = (PQ)(0) = \text{linear}((PQ)(1), \ldots, (PQ)(n))$
- Thus $xyz + s_1 + s_2 + s_3 = \text{linear}((PQ)(1) \cdot z, \ldots, (PQ)(n) \cdot z) + s_1 + s_2 + s_3$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P $\quad$ y, s_2, Q $\quad$ z, s_3

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$
- PQ is a degree-$(n-1)$ poly,
  $xy = (PQ)(0) = \text{linear}((PQ)(1), \ldots, (PQ)(n))$
- Thus $xyz + s_1 + s_2 + s_3 = \text{linear}((PQ)(1) \cdot z, \ldots, (PQ)(n) \cdot z) + s_1 + s_2 + s_3$
- Shamir Sharing: Safe to let i-th party know $P(i), Q(i)$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$
- PQ is a degree-$(n-1)$ poly,
  $xy = (PQ)(0) = \text{linear}((PQ)(1), \ldots, (PQ)(n))$
- Thus $xyz + s_1 + s_2 + s_3 = \text{linear}((PQ)(1) \cdot z, \ldots, (PQ)(n) \cdot z) + s_1 + s_2 + s_3$
- Shamir Sharing: Safe to let i-th party know $P(i), Q(i)$
  Magically, i-th party gets $P(i), Q(i)$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$
- PQ is a degree-$(n-1)$ poly,
  $xy = (PQ)(0) = \text{linear}((PQ)(1), \ldots, (PQ)(n))$
- Thus $xyz + s_1 + s_2 + s_3 = \text{linear}((PQ)(1) \cdot z, \ldots, (PQ)(n) \cdot z) + s_1 + s_2 + s_3$
- Shamir Sharing: Safe to let i-th party know $P(i), Q(i)$
  Magically, i-th party gets $P(i), Q(i)$, locally computes $(PQ)(i)$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

$x, s_1, \color{blue}{P}$	$y, s_2, \color{blue}{Q}$	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$
- PQ is a degree-$(n-1)$ poly,
  $xy = (PQ)(0) = \text{linear}((PQ)(1), \dots, (PQ)(n))$
- Thus $xyz + s_1 + s_2 + s_3 = \underbrace{\text{linear}((PQ)(1) \cdot z, \dots, (PQ)(n) \cdot z) + s_1 + s_2 + s_3}_{\text{degree 2 if } (PQ)(i) \text{ is locally computed}}$
- Shamir Sharing: Safe to let i-th party know $P(i), Q(i)$
  Magically, i-th party gets $P(i), Q(i)$, locally computes $(PQ)(i)$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

-  samples random degree-$\lfloor \frac{n-1}{2} \rfloor$ poly P, Q s.t. $P(0) = x, Q(0) = y$
- PQ is a degree-$(n-1)$ poly,
  $xy = (PQ)(0) = \text{linear}((PQ)(1), \ldots, (PQ)(n))$
- Thus $xyz + s_1 + s_2 + s_3 = \underbrace{\text{linear}((PQ)(1) \cdot z, \ldots, (PQ)(n) \cdot z) + s_1 + s_2 + s_3}_{\text{degree 2 if } (PQ)(i) \text{ is locally computed}}$
- Shamir Sharing: Safe to let i-th party know $P(i), Q(i)$
  ~~Magically, i-th party gets $P(i), Q(i)$, locally computes $(PQ)(i)$~~

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P | y, s_2, Q | z, s_3 | |
--- | --- | --- | --- | ---
$P(1), Q(1)$ | $P(2), Q(2)$ | $P(3), Q(3)$ | $P(4), Q(4)$ | $P(5), Q(5)$

- Thus $xyz + s_1 + s_2 + s_3 = \underbrace{\text{linear}((PQ)(1) \cdot z, \ldots, (PQ)(n) \cdot z) + s_1 + s_2 + s_3}_{\text{degree 2 if } (PQ)(i) \text{ is locally computed}}$

- Shamir Sharing: Safe to let i-th party know $P(i), Q(i)$
  ~~Magically, i-th party gets $P(i), Q(i)$, locally computes $(PQ)(i)$~~

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

- Thus $xyz + s_1 + s_2 + s_3 = \underbrace{\text{linear}((PQ)(1) \cdot z, \ldots, (PQ)(n) \cdot z) + s_1 + s_2 + s_3}_{\text{degree 2 if } (PQ)(i) \text{ is locally computed}}$
- Shamir Sharing: Safe to let i-th party know $P(i), Q(i)$
  ~~Magically, i-th party gets $P(i), Q(i)$, locally computes $(PQ)(i)$~~

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

-  holds $P(i)$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

-  holds $P(i)$
-  holds $Q(i)$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

-  holds $P(i)$
-  holds $Q(i)$
-  holds z

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

-  holds $P(i)$
-  holds $Q(i)$
-  holds z

new i-th party "gets" leakage $P(i), Q(i)$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

-  holds $P(i)$
-  holds $Q(i)$
-  holds z

new i-th party "gets" leakage $P(i), Q(i)$

formalized as "MPRE w/ leakage"

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

-  holds $P(i)$
-  holds $Q(i)$
-  holds z

new i-th party "gets" leakage $P(i), Q(i)$

formalized as "MPRE w/ leakage"

adversary & simulator gets $P(i), Q(i)$ if i-th party is corrupted

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

$$\text{function} = \det \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix}$$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

$$\text{function} = \det \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z + \text{some linear terms}$

$$\text{function} = \det \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix}$$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

$$\text{function} = \det\begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

$$\text{function} = \det \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

How to Sample $r_1, \ldots, r_5$?

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

$$\text{function} = \det \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

How to Sample $r_1, \ldots, r_5$? Let i-th party sample r_1, r_5

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

$$\text{function} = \det \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

How to Sample $r_1, \ldots, r_5$? Let i-th party sample r_1, r_5

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

$$\text{function} = \det \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

How to Sample $r_1, \ldots, r_5$? Let i-th party sample r_1, r_5 because $P(i), Q(i)$ can be leaked to i-th party

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P	y, s_2, Q	z, s_3		
$P(1), Q(1)$	$P(2), Q(2)$	$P(3), Q(3)$	$P(4), Q(4)$	$P(5), Q(5)$

NEW complete function: $P(i) \cdot Q(i) \cdot z +$ some linear terms

$$\text{function} = \det \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} P(i) & & \text{terms} \\ -1 & z & \\ & -1 & Q(i) \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix}$$

How to Handle degree-3 term? i-th party locally computes $r_1 r_5$

How to Sample $r_1, \ldots, r_5$? Let i-th party sample r_1, r_5 because $P(i), Q(i)$ can be leaked to i-th party

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P y, s_2, Q z, s_3

Putting-pieces-together

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P $\quad$ y, s_2, Q $\quad$ z, s_3

Putting-pieces-together

The encoding consists of

$$\begin{bmatrix} P(i)-r_1 & r_3P(i)+r_1z-r_1r_3-r_2 & r_1r_5z+r_4P(i)+r_2Q(i)-r_1r_4-r_2r_5+\text{terms} \\ -1 & z-r_3 & r_5z-r_4 \\ & -1 & Q(i)-r_5 \end{bmatrix} \text{for each } i.$$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P $\quad$ y, s_2, Q $\quad$ z, s_3

Putting-pieces-together

The encoding consists of

$$\begin{bmatrix} P(i)-r_1 & r_3P(i)+r_1z-r_1r_3-r_2 & r_1r_5z+r_4P(i)+r_2Q(i)-r_1r_4-r_2r_5+\text{terms} \\ -1 & z-r_3 & r_5z-r_4 \\ & -1 & Q(i)-r_5 \end{bmatrix} \text{ for each } i.$$

- , sample P, Q resp.

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P $\quad$ y, s_2, Q $\quad$ z, s_3

Putting-pieces-together

The encoding consists of

$$\begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix} \text{ for each } i.$$

- ,  sample P, Q resp.
- i-th party samples $r_{i,1}, r_{i,5}$ and locally compute $r_{i,1} r_{i,5}$

Direct Construction of degree-2 MPRE with Honest Majority

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1, P $\quad$ y, s_2, Q $\quad$ z, s_3

Putting-pieces-together

The encoding consists of

$$\begin{bmatrix} P(i) - r_1 & r_3 P(i) + r_1 z - r_1 r_3 - r_2 & r_1 r_5 z + r_4 P(i) + r_2 Q(i) - r_1 r_4 - r_2 r_5 + \text{terms} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & Q(i) - r_5 \end{bmatrix}$$ for each i.

-  sample P, Q resp.
- i-th party samples $r_{i,1}, r_{i,5}$ and locally compute $r_{i,1} r_{i,5}$
- $r_{i,2}, r_{i,3}, r_{i,4}$ are jointly sampled

NEXT

MPRE w/ OLE correlated randomness

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 y, s_2 z, s_3

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix}$$

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 y, s_2 z, s_3

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3x + r_1z \\ -r_1r_3 - r_2 \end{pmatrix} & \begin{pmatrix} r_1r_5z + r_4x + r_2y \\ -r_1r_4 - r_2r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ -r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} r_1 r_5 z + r_4 x + r_2 y \\ -r_1 r_4 - r_2 r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3x+r_1z \\ -r_1r_3-r_2 \end{pmatrix} & \begin{pmatrix} r_1r_5z+r_4x+r_2y \\ -r_1r_4-r_2r_5+s_1+s_2+s_3 \end{pmatrix} \\ -1 & z-r_3 & r_5z-r_4 \\ & -1 & y-r_5 \end{bmatrix}$$

How to Handle degree-3 term?

How to Sample $r_1, \ldots, r_5$?

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3x + r_1z \\ -r_1r_3 - r_2 \end{pmatrix} & \begin{pmatrix} r_1r_5z + r_4x + r_2y \\ -r_1r_4 - r_2r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

How to Sample $r_1, \dots, r_5$? Let  sample r_1.

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\qquad$ y, s_2 $\qquad$ z, s_3

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3x + r_1z \\ -r_1r_3 - r_2 \end{pmatrix} & \begin{pmatrix} r_1r_5z + r_4x + r_2y \\ -r_1r_4 - r_2r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

How to Sample $r_1, \ldots, r_5$? Let  sample r_1.

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ -r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} r_1 r_5 z + r_4 x + r_2 y \\ -r_1 r_4 - r_2 r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

How to Handle degree-3 term?

How to Sample $r_1, \ldots, r_5$? Let  sample r_1. Let  sample r_5.

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3x + r_1z \\ -r_1r_3 - r_2 \end{pmatrix} & \begin{pmatrix} r_1r_5z + r_4x + r_2y \\ -r_1r_4 - r_2r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

How to Handle degree-3 term? ,  sample r_1, r_5 from OLE Correlation!

How to Sample $r_1, \ldots, r_5$? Let  sample r_1. Let  sample r_5.

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ -r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} r_1 r_5 z + r_4 x + r_2 y \\ -r_1 r_4 - r_2 r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

How to Handle degree-3 term? (Alice), (Bob) sample r_1, r_5 from OLE Correlation!

How to Sample $r_1, \ldots, r_5$? Let (Alice) sample r_1. Let (Bob) sample r_5.

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

$r_1, a_1 \longleftrightarrow r_5, a_2$

s.t. $r_1 r_5 = a_1 + a_2$

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ -r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} r_1 r_5 z + r_4 x + r_2 y \\ -r_1 r_4 - r_2 r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

How to Handle degree-3 term? , sample r_1, r_5 from OLE Correlation!

How to Sample $r_1, \ldots, r_5$? Let sample r_1. Let sample r_5.

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

$r_1, a_1 \longleftrightarrow r_5, a_2$

s.t. $r_1 r_5 = a_1 + a_2$

$$\text{function} = \det \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \xrightarrow{\text{IK RE}} \begin{bmatrix} 1 & r_1 & r_2 \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x & & s_1+s_2+s_3 \\ -1 & z & \\ & -1 & y \end{bmatrix} \begin{bmatrix} 1 & r_3 & r_4 \\ & 1 & r_5 \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x - r_1 & \left(\begin{matrix} r_3 x + r_1 z \\ -r_1 r_3 - r_2 \end{matrix} \right) & \left(\begin{matrix} (a_1+a_2)z + r_4 x + r_2 y \\ -r_1 r_4 - r_2 r_5 + s_1 + s_2 + s_3 \end{matrix} \right) \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

How to Handle degree-3 term? , sample r_1, r_5 from OLE Correlation!

How to Sample $r_1, \ldots, r_5$? Let sample r_1. Let sample r_5.

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\quad$ y, s_2 $\quad$ z, s_3

r_1, a_1 $\longleftrightarrow$ r_5, a_2

s.t. $r_1 r_5 = a_1 + a_2$

In short

The encoding is

$$\begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ - r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} (a_1 + a_2) z + r_4 x + r_2 y \\ - r_1 r_4 - r_2 r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 y, s_2 z, s_3

r_1, a_1 $\leftrightarrow$ r_5, a_2

s.t. $r_1 r_5 = a_1 + a_2$

In short

The encoding is
$$\begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ -r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} (a_1 + a_2) z + r_4 x + r_2 y \\ -r_1 r_4 - r_2 r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

-  sample $(r_1, a_1), (r_5, a_2)$ from OLE correlated randomness

Direct Construction of degree-2 MPRE with OLE Correlations

MPRE for the complete function $xyz + s_1 + s_2 + s_3$

x, s_1 $\qquad$ y, s_2 $\qquad$ z, s_3

r_1, a_1 ⟷ r_5, a_2

s.t. $r_1 r_5 = a_1 + a_2$

In short

The encoding is

$$\begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ -r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} (a_1 + a_2) z + r_4 x + r_2 y \\ -r_1 r_4 - r_2 r_5 + s_1 + s_2 + s_3 \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

-  sample $(r_1, a_1), (r_5, a_2)$ from OLE correlated randomness
- r_2, r_3, r_4 are jointly sampled

Simple 2-round MPC

Our Results: information-theoretic 2-round MPC for NC1

(i) in plain model, w/ honest majority

(ii) in OLE correlation model, w/ honest minority

extension to P/poly with black-box use of PRG

new support arithmetic NC1 with black-box field access

new adaptive security with explicit simulator*

more efficient

Simple 2-round MPC

Our Results: information-theoretic 2-round MPC for NC1

(i) in plain model, w/ honest majority

(ii) in OLE correlation model, w/ honest minority

extension to P/poly with black-box use of PRG

new support arithmetic NC1 with black-box field access

new adaptive security with explicit simulator*

more efficient

Technique:

a new **direct** construction of degree-2 MPRE with no round collapsing

$$xyz + \text{terms} = \det \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ - r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} \boxed{r_1 r_5 z} + r_4 x + r_2 y \\ - r_1 r_4 - r_2 r_5 + \text{terms} \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

Simple 2-round MPC

Our Results: information-theoretic 2-round MPC for NC1

(i) in plain model, w/ honest majority

(ii) in OLE correlation model, w/ honest minority

extension to P/poly with black-box use of PRG

new support arithmetic NC1 with black-box field access

new adaptive security with explicit simulator*

more efficient

Technique:

a new **direct** construction of degree-2 MPRE with no round collapsing

$$xyz + \text{terms} = \det \begin{bmatrix} x - r_1 & \begin{pmatrix} r_3 x + r_1 z \\ - r_1 r_3 - r_2 \end{pmatrix} & \begin{pmatrix} \boxed{r_1 r_5 z} + r_4 x + r_2 y \\ - r_1 r_4 - r_2 r_5 + \text{terms} \end{pmatrix} \\ -1 & z - r_3 & r_5 z - r_4 \\ & -1 & y - r_5 \end{bmatrix}$$

Thank you!

"So simple that can be taught in class."